MATHEMATICAL MODEL OF THE DOUBLE EFFECT TELESCOPIC HYDRAULIC DAMPER

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ABSTRACT

The paper presents the fundamental aspects of the double effect telescopic hydraulic damper used by rail vehicles. A mathematical model that may helps to the realization of some numerical simulation is also proposed. The results of the simulations are compared to experimental data.

KEYWORDS

telescopic hydraulic damper, mathematical model, numerical simulation

NOMENCLATURE

\( F \) force at the hydraulic cylinder rod
\( m \) reduced mass
\( F_f \) friction force
\( p_1, p_2 \) cylinder rooms pressure
\( A_1, A_2 \) cylinder room area
\( \Delta p_{dr} \) throttle pressure losses
\( \Delta p_{v2} \) valve pressure losses
\( \rho \) oil density
\( v_p \) piston velocity
\( v_s \) valve flow velocity
\( \nu \) oil kinematic viscosity
\( \lambda \) throttle flow linear loss coefficient
\( \xi \) throttle flow local loss coefficient
\( a_1, a_2, l_1, l_2 \) throttle geometric characteristics

ABBREVIATIONS

AHT – hydraulic telescopic damper

1. INTRODUCTION

The hydraulic dampers performances have a major influence both in vehicle stability and the passengers’ comfort. In most cases, the design objective is a compromise between comfort and stability. Hard dampers are used for vehicles that must be able to run on very uneven roads. These dampers diminish the comfort, but increase the adherence to the road.

The hydraulic damper has a double effect and it suppresses the oscillations in both senses with a greater energy in spring relaxation phase.

The hydraulic telescopic dampers with two tubes represent the most used constructive solution. The oleo-pneumatic damper represents a progress in dampers realization.

2. GENERAL CHARACTERISTICS AND CONSTRUCTIVE ELEMENTS

A bitubular hydraulic telescopic damper transforms the kinetic energy of the oscillating movement in thermic energy dissipated outwards. This energy transformation is the result of the viscous friction that appears at the damper fluid passing through the calibrated lamination orifices.

The AT 13 AHT, used at the wagons, schematic diagram is presented in Figure 1.

In compression phase, the piston 10 realizes an over-pressure in lower room. The valve 9 is opened and a connection with upper room is realized. When the force \( F \) exceeds an imposed limit, the valve 8 is opened too.

In relaxation phase, the piston 10 compresses the oil from upper room of the cylinder 6. The valve 9 is closed, and the oil from the tank 12 is aspiried in lower room through valve 11. The oil is evacuated through
Figure 1. The constructive scheme of AHT damper throttle 2, and when the pressure exceeds a limit imposed by the force $F$ value, the valve 8 is opened too.

The oil evacuation is realized through the same circuit composed from throttle 2 and valve 8, no matter which phase takes place.

The volume and the oil flow differences between the two phases leads to an energy difference dissipated for compression and relaxation, respectively.

3. AHT DAMPER MATHEMATICAL MODEL

The relaxation damping force $F_d$ and the compression damping force $F_c$ represent the resistances realized by the damper when the piston run the relaxation stroke and compression stroke, respectively. The diagram of these forces versus the piston velocity $v_p$ represents the damper external characteristic.

3.1. Compression phase

The Figure 2 represents the hydraulic scheme of the AT 13 damper in compression phase. This scheme highlights the fluid circuit.

The mathematical model results from the system analytical description:

1. Dynamic balance equation

$$F_c = m \frac{dv_p}{dt} + F_f + p_1 A_1 - p_2 a_2$$

2. Bernoulli equations for sections 1-2, and 2-3 respectively

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \frac{p_2}{\gamma} + \frac{\Delta p_{12}}{\gamma} + \frac{\alpha \cdot v_2^2}{2g} + \frac{l \cdot dv_2}{dt}$$

and

$$\frac{p_2}{\gamma} + \frac{\alpha \cdot v_2}{2g} = \frac{p_2}{\gamma} + \frac{\alpha \cdot v_2^2}{2g}$$

Taking into consideration the following values: $\alpha = 1$, $l = 0$, $\Delta p_{12} = p_2$ and $v_1 = v_{i2}$, from Eq. (2) and (3), it obtains:

$$p_1 = \Delta p_{12} + \Delta p_{12}$$

$$p_2 = \Delta p_{12}$$

3. The fluid continuity equation:

$$Q = Q_1 + Q_2$$

where: $Q = A_1 \cdot v_p$, $Q_1 = A_2 \cdot v_p$.

The fluid flow through throttle is:

$$Q_2 = Q - Q_1 = v_p (A_1 - A_2)$$

4. Pressure losses through throttle and valve, respectively:

$$\Delta p_{12} = q \left( \frac{Q_2}{\mu \cdot A_{12}} \right)^{2}$$

$$\Delta p_{12} = q \left( \frac{Q}{\mu \cdot A_{12}} \right)^{2}$$

5. It will be considered the friction force $F_f$ through $\delta$ space piston – cylinder under laminar hypothesis:

$$F_f = v \cdot \rho \cdot \pi D \cdot b \cdot \frac{v_p}{\delta}$$
From Eqs. (1) –(10) it results that:

$$F_c = m \frac{dv_p}{dt} + \frac{\nu \pi Db}{\delta} v_p + (A_1 - A_2)p_2 \quad (11)$$

Taking into consideration, the throttle geometry, it results:

$$F_c = m \frac{dv_p}{dt} + \frac{\nu \pi Db}{\delta} v_p + \frac{\rho}{2} (A_1 - A_2)^3 k(Re)v_p^2 \quad (12)$$

where

$$k(Re) = \frac{1}{a_2^2} \left(1 + \frac{\xi}{\lambda_2} \frac{l_2}{d_2^2} \right) + \frac{1}{a_1^2} \left(\lambda_1 \frac{l_1}{d_1^2} - 1 \right) \quad (13)$$

3.2. Relaxation phase

The relaxation phase corresponding hydraulic scheme is presented in Figure 3.

![Figure 3. Relaxation phase](image)

The equation for the relaxation force $F_d$ results following the same calculus procedure. So,

$$F_d = m \frac{dv_p}{dt} + \frac{\nu \pi Db}{\delta} v_p + \frac{\rho}{2} (A_1 - A_2)^3 k(Re)v_p^2 + \Delta p_{db} A_2 + \Delta p_{sl} A_1 \quad (14)$$

The following hypothesis is taking into consideration: $\Delta p_{sl} \equiv 0$ and $\Delta p_{db} = p_2$ and it results:

$$F_d = m \frac{dv_p}{dt} + \frac{\nu \pi Db}{\delta} v_p + \frac{\rho}{2} (A_1 - A_2)^3 k(Re)v_p^2 + p_2 A_2 \quad (15)$$

where $p_2 = \frac{\rho}{2} (A_1 - A_2)^2 k(Re)v_p^2$ and $k(Re)$ is defined in Eq. (13).

The relaxation force $F_d$ is defined in Eq. (16) as:

$$F_d = m \frac{dv_p}{dt} + \frac{\nu \pi Db}{\delta} v_p + \frac{\rho}{2} (A_1 - A_2) \left[1 + (A_1 - A_2) A_2 k(Re) v_p^2 \right] \quad (16)$$

3.3. Simulation diagram

The diagram used to simulate the AHT damper is presented in Figure 4. The simulation was made using the MATLAB – Simulink software.

The simulation was made taking into consideration the fact that the piston movement is a sinusoidal wave that has the form:

$$s(t) = s_0 \sin(\omega t)$$

where $s_0$ is the maximum piston displacement and $\omega$ is the wave angular frequency.

![Figure 4. Simulation diagram.](image)

![Figure 5. Force and piston velocity time history](image)
4. NUMERICAL RESULTS

The simulation results were obtained taking into consideration the following values for the parameters that appear in Eq. (1) – (16)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\omega$</td>
<td>9.52 [rad/s]</td>
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<tr>
<td>$\lambda_1$</td>
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</tr>
<tr>
<td>$\lambda_2$</td>
<td>0.03</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$m$</td>
<td>10 [kg]</td>
</tr>
<tr>
<td>$\nu$</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>940 [kg/m$^3$]</td>
</tr>
<tr>
<td>$D$</td>
<td>0.041 [m]</td>
</tr>
<tr>
<td>$b$</td>
<td>0.03 [m]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$4 \times 10^{-4}$ [m]</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The paper presents the damper as a hydraulic system. The schemes for compression phase and for relaxation phase are presented.

A mathematical model and a simulation scheme using MATLAB Simulink are obtained.

The AT 13 damper external characteristic was obtained by simulation. The damper is supposed working in a dynamic regime.

The characteristics obtained by simulation are confirmed by experimental results.

REFERENCES